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Quasi-static dislocations in three dimensional inhomogeneous media

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Abstract. We present an approximate (perturbation) method for computing surface displacement fields due to quasi-static dislocations in inhomogeneous elastic half-spaces, extending the work of Du et al. [1994] to three dimensions. Three dimensional methods are required to model geodetic observations. The perturbation method allows one to include more realistic earth structure in the analysis of surface deformation measurements, and is computationally efficient enough to be used in inversions for source properties. As an example, we investigate the effect of material inhomogeneity on the location of the fault plane for the 1989 Kalapana, Hawaii, earthquake. Inversion of coseismic elevation changes assuming a homogeneous half-space yielded a finite source depth markedly shallower than the mainshock and principal aftershocks [Árnadóttir et al., 1991]. Incorporation of lateral and vertical variations in shear modulus, consistent with velocity and density data, yields a source depth more consistent with the seismic results. Failure to account for variations in elastic properties can bias estimates of source depth and moment.

Introduction

Spatial variations in elastic properties are known to have a significant influence on surface displacement fields [*Rybicki,* 1971; *Savage,* 1987; *Roth,* 1990]. Despite this, uniform half-space models are widely used to analyze crustal deformation measurements. This situation exists because there have not been simple and efficient methods for computing elastic fields due to dislocations in inhomogeneous media. Exact analytical methods are often very difficult to extend to the three dimensional geometries needed to model actual geodetic observations from all but the largest earthquakes. Numerical procedures, such as finite elements or boundary element methods, are computationally intensive and thus not well suited to inversion of geodetic data for fault parameters, where many forward computations are required. Yet, as the quality of geodetic observations improve and as we seek more detailed information about source properties it will become increasingly important for models to account for realistic earth structure. For example, studies of the 1989 Kalapana, Hawaii, and Loma Prieta, California, earthquakes suggest that failure to account for spatial variation in elastic properties can result in different finite source geometries inferred from geodetic and seismic observations [*Árnadóttir et al.,* 1991; *Eberhart-Phillips and Stuart,* 1992]. *Du et al.* [1994] presented a moduli perturbation method for computing surface displacements due to dislocations in inhomogeneous media and presented some two- dimensional solutions. In this paper, we extend the method to three-dimensions. As an example, we investigate the effect of material inhomogeneity on the location of the fault plane for the 1989 Kalapana, Hawaii, earthquake.

Moduli Perturbation Method

The general formulation of the moduli perturbation approach is given in *Du et al.* [1994]. In this procedure, a homogeneous half-space with shear modulus μ_0 yields the zero-order solution. If the medium has piece-wise constant variations in shear modulus, the first order correction in the displacement $u_m^{(1)}(\mathbf{x})$ is given by

$$u_m^{(1)}(\mathbf{x}) = -\frac{\delta\mu}{\mu_0}\int_S \sigma_{ij}^{(0)}(\xi)G_{mi}(\mathbf{x},\xi)n_j ds(\xi) \qquad (1)$$

where, $\delta\mu$ is the difference between the shear modulus and that in the reference state, $\sigma_{ij}^{(0)}$ is the stress from the zero order solution (that is the stress due to a dislocation in a homogeneous half-space), and $G_{mi}(\mathbf{x},\xi)$ is the elastic Green's function (displacement at $\mathbf{x}$ due to point force in the i-th direction at $\boldsymbol{\xi}$) in a homogeneous half-space. The integral is over the interface between the two different materials, and n_j is the unit normal to the interface. Multiple boundaries are easily treated by superposition. In order to guarantee convergence of the perturbation expansion, we choose the reference modulus μ_0 so that $\delta\mu/\mu_0 < 1$.

To test the numerical procedures used here, we compare the perturbation result to an analytical solution for a dilatational point source in a half-space beneath a layer [*Ben-Menahem and Gillon,* 1970]. The layer has thickness H and shear modulus μ_1, while the underlying half space has shear modulus μ_2. A point source with intensity M_0 is located at depth C. For simplicity, we evaluate the vertical displacement only, which at the free surface in a homogeneous half space is

$$u_3^{(0)}(x,y,0) = \frac{M_0(1-2\nu)C}{2\pi\mu_2 R^3} \qquad (2)$$

[*Okada,* 1992], where $R = \sqrt{x^2+y^2+C^2}$. Note that

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we have chosen the substrate for the zeroth-order solution since the surface layer is more compliant. From (1), the first-order correction is

$$u_3^{(1)}(x,y,0) = \frac{\delta\mu}{\mu_2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\sigma_{13}^{(0)}G_{31}+\sigma_{23}^{(0)}G_{32} + \sigma_{33}^{(0)}G_{33}]\,dx'\,dy' \quad (3)$$

where $\delta\mu = \mu_1 - \mu_2$, G_{ij} are the Green's functions for a homogeneous half space, and $\sigma_{ij}^{(0)}$ are the stresses due to a dilatational point source in a homogeneous half space [*Okada,* 1992]. To efficiently evaluate the double integral in (3), we transform the infinite domain of integration onto a unit square using $s = \frac{2}{\pi}tan^{-1}(x')$ and $t = \frac{2}{\pi}tan^{-1}(y')$. Equation (3) becomes

$$u_3^{(1)}(x,y,0) = \frac{\delta\mu\,\pi^2}{4\mu_2}\times \int_{-1}^{+1}\int_{-1}^{+1}[\sigma_{13}^{(0)}(s,t)G_{31}(s,t)+\sigma_{23}^{(0)}(s,t)G_{32}(s,t) + \sigma_{33}^{(0)}(s,t)G_{33}(s,t)]\; sec^2(\frac{\pi s}{2})\; sec^2(\frac{\pi t}{2})\; dt\; ds \quad (4)$$

The double integral in (4) is numerically evaluated using the 61-point Konrod rule obtained by optimal addition of abscissae to the 30-point Gauss rule [*Piessens et al.,* 1983]. If we consider the surface vertical displacement at the point right above the dilatational source for a special case of $C = 2H$ and $\nu = 0.25$, we are able to obtain a closed-form solution for the first-order correction

$$u_3^{(1)}(0,0,0) = \frac{7(1-\gamma)M_0}{384\pi\mu_2 H^2} \quad (5)$$

where $\gamma = \mu_1/\mu_2$. Thus, the net displacement $u_3^{(0)}+u_3^{(1)}$ is

$$u_3(0,0,0) = \frac{M_0}{16\pi\mu_2 H^2}\left[1+\frac{7}{24}(1-\gamma)\right] \quad (6)$$

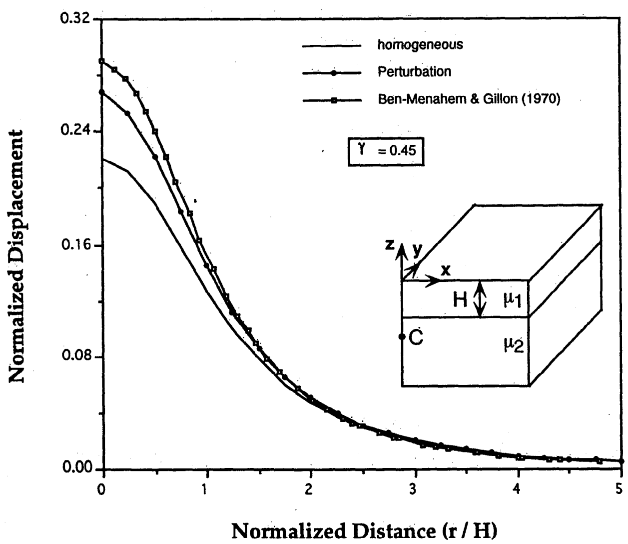

Figure 1. Vertical displacements at $z = 0$ due to a dilatational point source beneath a layer of thickness H. The displacements are normalized by $M_0/2\pi H^2$. Displacement is shown as a function of the normalized distance from the source r/H for the first-order perturbation solution and the exact solution of Ben-Menahem and Gillon [1970] for $\gamma = 0.45$, and $C = 1.5H$.

Comparisons of the closed-form solution (6) and numerical integration of (4) indicate that relative numerical errors are in the order of 10^{-4}, validating the numerical procedure. In Figure 1 we show that the first-order perturbation solution agrees reasonably well with the analytical solution of *Ben-Menahem and Gillon* [1970], for $C = 1.5H$ and $\gamma = 0.45$. The relative error in the perturbation approximation nowhere exceeds 7.5%.

Effect Of Inhomogeneity On Depth Of The 1989 Kalapana Earthquake

As an example of the effect of inhomogeneity on geodetic displacements we consider the 1989 Kalapana earthquake (M6.1) which occurred below the south flank of Kilauea volcano, Hawaii. The main shock focal depth determined by the U.S.G.S. Hawaiian Volcano Observatory is $\sim$ 9 km. Aftershocks were concentrated within a nearly horizontal zone from 6 to 9 km deep (Figure 2). Long period seismic and geodetic data indicate that the 1989 earthquake involved slip on a subhorizontal fault with motion of the upper block directed southward away from the East Rift Zone [*Chen et al.,* 1990; *Árnadóttir,* 1991; *Dvorak,* 1994], although the p-wave first motions indicate that rupture initiated on a moderately dipping plane [*Bryan,* 1992].

Vertical coseismic displacements from repeated leveling were presented by *Árnadóttir et al.* [1991]. GPS and other measurements are not analyzed here for brevity. Our purpose is to show the effect of non-uniform properties on inversions for source geometry, rather than to obtain the solution best fitting all of the data. *Árnadóttir et al.* [1991] showed that, assuming a homogeneous elastic half space, the data could be well fit by a flat-lying dislocation with uniform slip at a depth of 4 ± 1, two to five kilometers shallower than the mainshock and its well located aftershocks (Figure 2). They noted that a compliant surface layer, as inferred from seismic velocity determinations [*Klein,* 1981], could cause the dislocation modeled in a homogeneous half space to appear shallower than it really is. *Du et al.* [1994] noted that the core of the east rift zone exhibits high densities and velocities [*Hill and Zucca,* 1987] and is likely to be less compliant than the adjacent flank. They suggested that a combination of vertical and horizontal variations in the elastic properties might be necessary to properly model the volcano structure. However, because of the inherently three-dimensional nature of the 1989 earthquake, the two-dimensional results of *Árnadóttir et al.* [1991] and *Du et al.* [1994] cannot be directly compared with the leveling data.

Shallow basalt lavas with low seismic velocities are represented by a compliant horizontal layer with thickness 5 km and modulus μ_1 (Figure 3) overlaying a stiffer horizontal substrate (modulus μ_2). The east rift zone is represented by a stiff vertical slab (μ_3). We follow *Árnadóttir et al.* [1991] and take the ratio of shear modulus of the surface compliant layer to that of the stiffer substrate in the south flank (μ_1/μ_2) to be 0.34 based on seismic velocity and density models [*Hill and Zucca,* 1987; *Klein,* 1981]. The average P-wave velocity and density in the upper few kilometers of the south flank of Kilauea are roughly 3–3.5 km/s and 2.6–2.7 g/cm^3, while the average P-wave velocity and density in the rift zone are about 7.1 km/s and 2.9 g/cm^3 [*Hill and Zucca,*

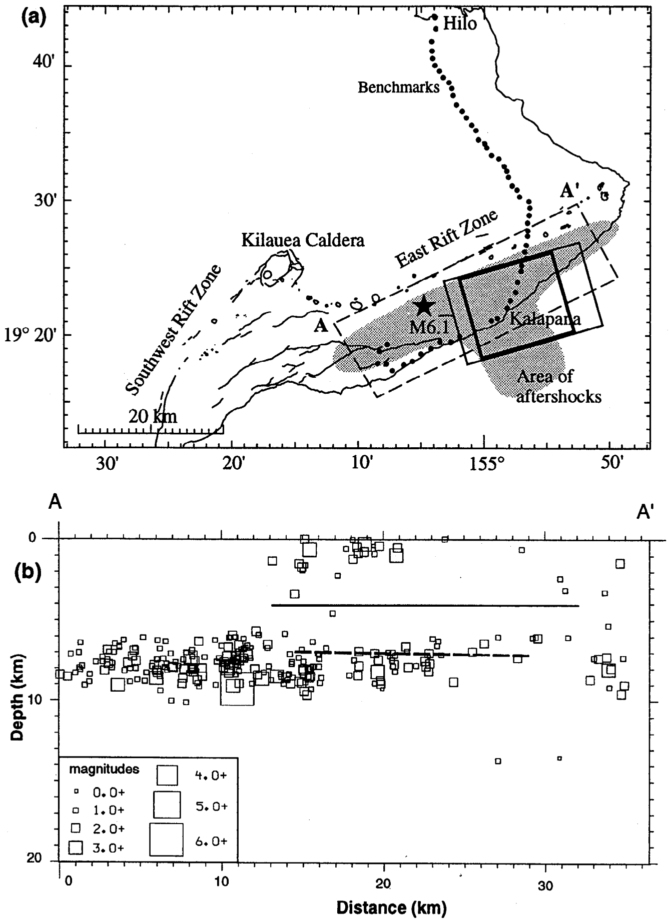

Figure 2. (a) Map of level line surveyed before and after the 1989 Kalapana, Hawaii earthquake. The location of the main shock epicenter is shown by a star, the aftershock zone by the shaded region. The surface projection of the dislocation in a homogeneous medium that best fits the leveling data is shown by the thin solid rectangle [*Árnadóttir et al.,* 1991]. The heavy solid rectangle shows the projection of the dislocation estimated here including both vertical and lateral variations in shear modulus. The dashed rectangle indicates the location of the cross section shown in Figure 2b. (b) Aftershocks of the 1989 Kalapana earthquake projected onto a vertical plane A - A'. The aftershocks cover a period from June 24, 1989 to September 1, 1989. The location of the dislocation surface estimated from the inversion of the leveling data assuming a uniform half-space is shown by the solid line. The dashed line denotes the location of dislocation surface estimated here including vertical and lateral variations in shear modulus. Largest square is mainshock.

1987; *Klein,* 1981]. Thus, $\mu_1 \sim 10$ GPa, $\mu_2 \sim 29$ GPa, $\mu_3 \sim 50$ GPa and $\mu_1/\mu_3 = 0.2$.

In order to determine the fault parameters that produce the best fit to the data in a least-squares sense, we use a Levenberg-Marquardt nonlinear least squares optimization algorithm in MATLAB [*Grace,* 1994]. We include the correlations between adjacent data, that are inherent in leveling observations [e.g., *Árnadóttir et al.,* 1992].

Nine parameters are required to describe a rectangular dislocation. Based on the results of *Árnadóttir et al.* [1991] the inferred fault plane is nearly horizontal, so that the fault strike is not well constrained. We thus simplify the problem by fixing the strike of the fault to be that of the best fitting homogeneous model. We also note from the homogeneous model that the strike-slip component of the fault slip is negligible. We thus constrain the strike-slip to zero, further reducing the number of model parameters from 9 to 7.

The results of the optimization indicate that the best-fitting fault model has a depth of 7 km, length of 13 km, width of 11 km, and a dip angle of 3.7° to the southeast, and slip is ~ 1.5 m. The distance between the southeast boundary of the rift zone and the northwestern tip of the fault is 1 km. The geodetic moment, $M_0 = \mu A s$ is 6×10^{18} Nm, where A is fault area, s is slip, and $\mu = 29$ GPa. This is 15% larger than the double-couple moment of 5.2×10^{18} Nm estimated from the CMT solution for the earthquake [*National Earthquake Information Center,* 1989], and 25% less than the geodetic moment of 8×10^{18} Nm estimated from the inversion of the same set of leveling data in a homogeneous medium by *Árnadóttir et al.* [1991]. Figure 4 shows the vertical displacements for the best-fitting model, in comparison to the data, and the vertical displacements due to the same fault in a homogeneous half space.

It is difficult to determine confidence intervals in the parameters for non-linear optimization problems. Realistic uncertainties in the seismically inferred depths can also be problematic, as many estimates are conditional on the assumed velocity structure. Approximate 95% confidence bounds on the fault depth were computed using an F-test [see for example, *Árnadóttir and Segall,*

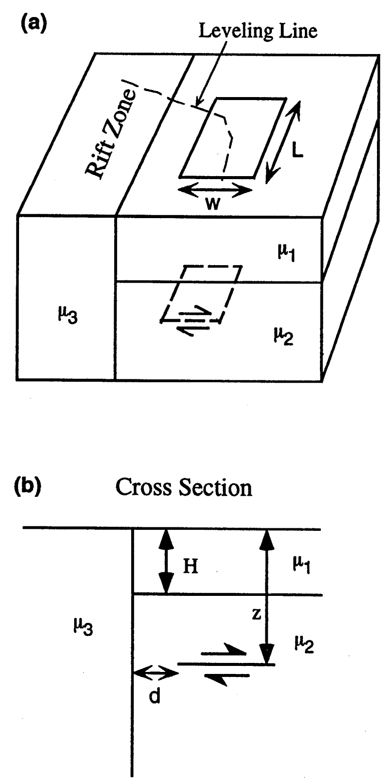

Figure 3. (a) A 3-D sketch and (b) a cross section perpendicular to the rift zone, showing the geometrical configuration of a horizontal fault in an inhomogeneous half space for the 1989 Kalapana, Hawaii, earthquake.

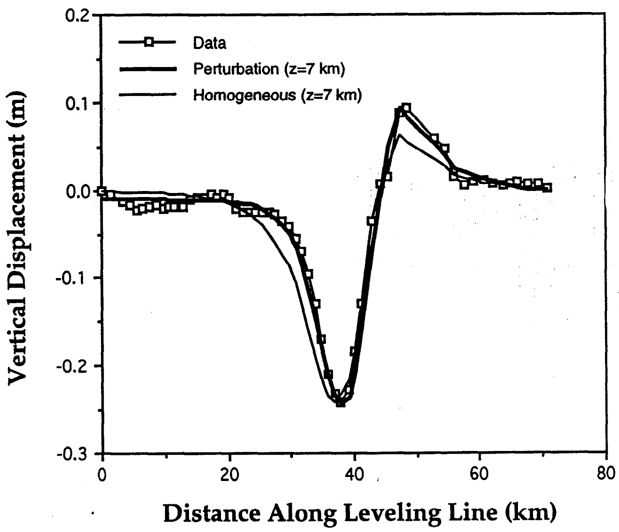

Figure 4. Comparison of vertical displacements due to a fault at 7 km depth in an inhomogeneous half space with both vertical and lateral variations in shear modulus with the observed data, and with those due to the same fault in a homogeneous half space.

1994]. According to this procedure, fault depths in the range of 6.25 to 7.5 km provide an acceptable fit to the data, at the 95% confidence level, which overlaps the aftershock zone at 6 to 9 km (Figure 2).

While we have analyzed only a subset of the available data, it appears that a three-dimensional model with both a shallow compliant horizontal layer in the south flank and a stiff vertical east rift zone can help to explain the discrepancy between geodetic and seismic fault depths for the 1989 Kalapana, Hawaii, earthquake. We note from Figure 1 that the mainshock epicenter is off the western end of the modeled fault. This presumably indicates that slip tapered to the west and that some slip occurred outside the modeled uniform slip dislocation, which is a common feature of such inversions [e.g., *Árnadóttir and Segall,* 1994]. It is also not unusual for aftershocks to be concentrated near the periphery of the high slip zone.

Conclusions

We have extended the moduli perturbation method of *Du et al.,* [1994] to the three- dimensional inhomogeneous half space. This provides a computationally efficient method for including more realistic earth structure in computing the quasi-static deformation fields due to earthquake sources. As an example, we investigated the effect of material inhomogeneity on the location of the fault plane for the 1989 Kalapana, Hawaii. In a homogeneous half-space the best-fitting dislocation lies at a depth of 4 km. Inclusion of lateral and depth dependent variations in shear modulus inferred from seismic and gravity data, increases the fault depth to $\sim$ 7 km, more consistent with the 6-9 km depth of the aftershock zone. For this particular geometry, failure to account for spatially varying elastic properties can cause one to underestimate source depth and overestimate the seismic moment.

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